8310 HOMEWORK 3. (and my appreciation to John Drake for letting steal, with modification, one of his problems)

The goal is to work with some basic evolutionary principles to help cement the concepts.

In 1 and 2, we'll let *Biston betula* provide the context. Assume that our system is diploid, undergoes sexual reproduction, has a very large (infinite) population, matings are random: i.e., Hardy-Weinberg equilibrium can be assumed. Also assume that the gene that determine color morph has two alleles (A and a), with A (dark morph allele) dominant to a (the light morph allele).

1) Kettlwell (1956) reported that the *carbonaria* phenotype (the dark morph) comprised 87% of all moths collected at a site near Birmingham. Assuming Hardy-Weinberg equilibrium, calculate: a) the frequency of the dominant allele; and b) the proportion of the *carbonaria* moths that are heterozygotes.

a)

$$Darkfreq = .87 = p^2 + 2pq$$
$$Lightfreq = .13 = q^2$$
$$\therefore q = \sqrt{.13} = .36$$
$$p = 1 - q = .64$$

b)

$$FreqDarkHomoz = p^2 = .64^2 = .41$$
$$FreqDarkHeteroz = 2pq = .46$$
$$FreqLightHomoz = q^2 = .13$$

Thus, the frequency of the dark morphs that are heterozygotes is .46/.41+.46 = .529

2) Now, assume the allelic frequencies are 0.8 (dark allele) and 0.2 (light allele) and that there is selection against the dark morph (the relative fitness of the dark morph is 0.6 and the relative fitness of the light morph is 1.0). Assume that the differences in fitness are based entirely on survival up to the age of reproduction. Find: a) the frequency of the two *morphs* AFTER selection (if the allelic frequencies before selection were .8 and .2); b) the frequency of three *genotypes* AFTER selection; c) the frequency of the two *morphs* at the start of the next generation; d) if you continued this process for many generations (assuming the environment remains constant) what will be the eventual frequency of the dark morph and of the light morph? e) give two reasons why the dark allele might persist in the system?

a) Let's track the three genotypes:

| Genotype | Morph | Before selection | After selection | Adjust to sum to 1 |
|---|---|---|---|---|
| AA | dark | .8x.8=.64 | .64x.6=.384 | .382/.616=.623 |
| Aa | dark | 2x.8x.2=.32 | .32x.6=.192 | .192/.616=.312 |
| aa | light | .2x.2=.04 | .04x1=.04 | .04/.616=.065 |

Therefore the dark morph comprises 93.5% of the population after selection (.623+.312) and the light morph comprises 6.5%.

b) the frequency of the three genotypes is given above: .623, .312 and .065

c) Assuming random mating, then these surviving adults give rise to the next generation according to HW:

Find find p and q in the breeding adult population:
p = (2x.623 + .312) / 2 = .779
q = (.312 + 2x.065) /2 = .221

So, now we can find the offspring genotype frequencies via $p^2$, 2pq and$q^2$:

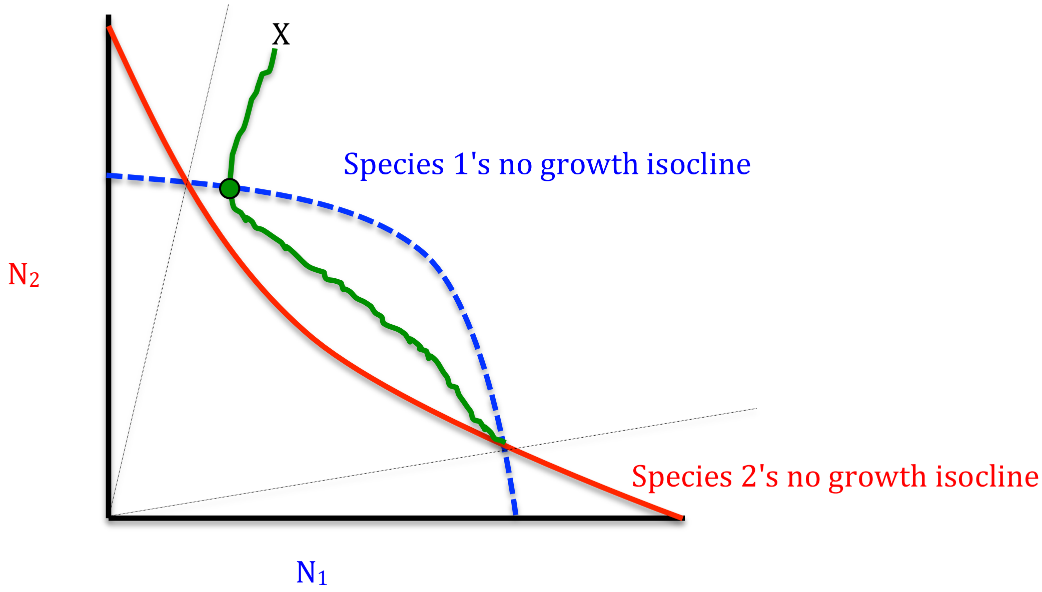
p2=.607
2pq=.344
q2=.049

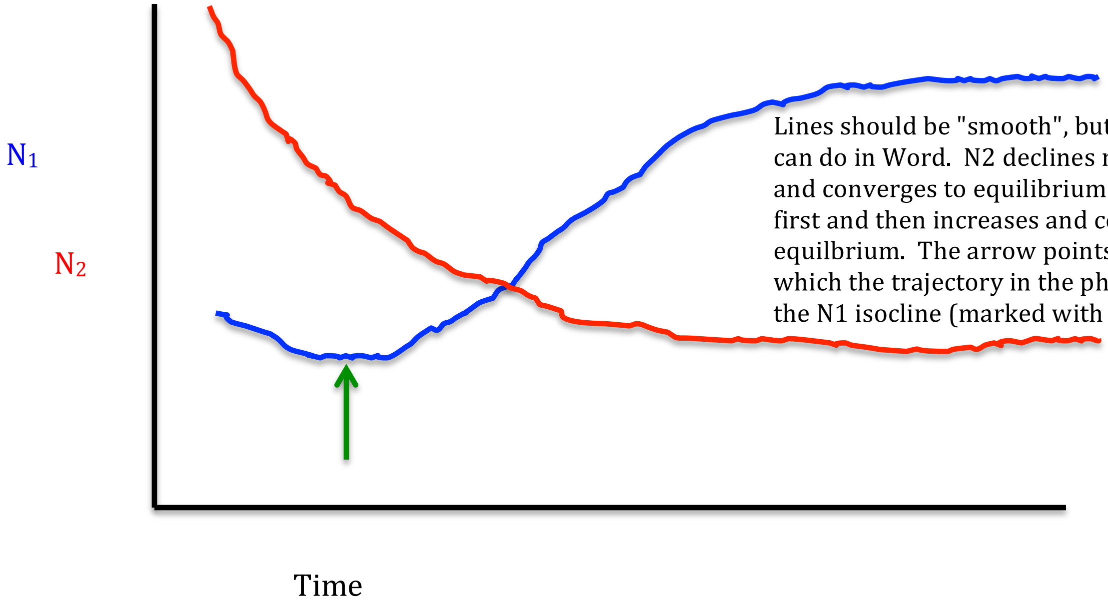
Thus, the frequency of the Dark morph is .607+.344 = .951
The frequency of the light morph is .049

d) In the absence of any other processes, and assuming the environment remains constant, the q allele will eventually go to fixation. Thus, the population will be 100% light morph.

e) e.g., mutation from light to dark; changing environment; immigration from sites with non-light morph; a trade-off with other fitness components (although this would change the selection coefficient that I first gave you).

3) For the following phase-plane for two *competitors* (note the non-linearities), please a) sketch the vector field on the phase plane (i.e., draw the initial trajectories of the system for a bunch of different density combinations); and b) for the point marked 'X', draw the ensuing dynamics, both as a trajectory on the phase plane AND in a plot of N vs. time. The thin black dashed lines are manifolds (separatrix lines).





Lines should be "smooth", but this is the best I can do in Word. N2 declines monotonically and converges to equilibrium. N1 declines at first and then increases and converges to equlibrium. The arrow points to the time at which the trajectory in the phase-plane crosses the N1 isocline (marked with a green dot).