

# Parasitism

## Key concepts

- Immunizing and non-immunizing pathogens
- *SI*, *SIS*, and *SIR* epidemics
- Basic reproduction number,  $R_0$

## Tasmanian devil facial tumor disease

The Tasmanian devil *Sarcophilus harrisii* is a medium-sized (6-8 kg), non-territorial, solitary carnivorous marsupial, once widespread in Australia but restricted to Tasmania (and some nearby islands) for around the last 3000 years. It is aggressive and may prey on species much larger in size than itself. Competition among males for mates may also lead to aggressive encounters. In early modern times, Tasmanian devils were hunted both as a perceived threat to livestock and for fur, but have been protected since the middle 20th century.

Tasmanian devils have been recorded to experienced severe declines on at two occasions in the 20th century. In the mid-1990s, the population of Tasmanian Devils was estimated to comprise more than 100,000 individuals. However, since that time an infectious cancer – Tasmanian devil facial tumor disease (DFTD) – has resulted in severe declines so that by 2008 less than 15,000 individuals were believed to exist in the wild. DFTD is a transmissible cancer that is passed during aggressive encounters. Affected populations suffer high disease induced mortality, which may approach 100% in high density populations. Symptoms of infection include the growth of facial lesions that develop into tumors which eventually interfere with feeding and lead to death by starvation. Due to past population bottlenecks, genetic diversity of Tasmanian devils is low so that the species is both unusually susceptible



Figure 1: Tasmanian devil *Sarcophilus harrisii*. Image by W. McLean (source: Wikipedia).

to infection and lacking a gene pools from which resistance might develop. Extinction in the wild is predicted to occur by 2035.

## Non-immunizing infections

The physiological and epidemiological consequences of parasitism vary widely. For theoretical purposes, one typically distinguishes *pathogens* (also called microparasites) from macroparasites. Pathogens are those parasites that cause an effectively binary infection status in the host (infected/uninfected), including viruses, bacteria, fungi, and the unusual case of DFTD. In macroparasites, such as gastrointestinal flatworms and arthropod ectoparasites, both severity of disease and transmission are related to infection burden. The remainder of this chapter deals with the dynamics of pathogens.

The population of hosts of a non-immunizing infectious pathogens, such as DFTD, may be divided into two compartments, with abundance of susceptible individuals denoted by  $S$  and abundance of infected individuals denoted by  $I$ . By assumption, the total population size is  $S+I = N$ . The *force of infection*, denoted  $\lambda$  refers to the per capita rate at which susceptible individuals contract the infection. Thus, the dynamics of the susceptible population obey

$$\frac{dS}{dt} = b(N)N - dS - \lambda S, \quad (1)$$

where  $b(N)$  is the per capita birth rate,  $d$  is the natural death rate, and the negative sign on  $\lambda$  indicates that this is a flux out of the susceptible pool of individuals. All individuals acquiring infection enter the infected class so the dynamics of the infected class are given by

$$\frac{dI}{dt} = \lambda S - dI - \alpha I, \quad (2)$$

where  $\alpha$  is the rate of disease induced mortality. In general, the force of infection is a function of the number of infected individuals in the population. When this function is simply linear,

$$\lambda(I) = \beta I, \quad (3)$$

the transmission is called *density-dependent* and the coefficient of proportionality  $\beta > 0$  is called the *transmissibility*. Density-dependent transmission occurs when individual exposure increases with the number of individuals in the population. An alternative is that the force of infection may be proportional to the *proportion* of infected individuals in the population. In this case, transmission is called *frequency dependent* and the force of infection is

$$\lambda(I) = \beta \frac{I}{N}. \quad (4)$$

In cases where population size does not change (*i.e.*,  $N$  is a fixed constant), the only difference between density-dependent and frequency-dependent transmission is the interpretation of the transmissibility parameter. The two models are numerically equivalent. However, if the pathogen causes some disease induced mortality or population size is changing for other reasons (*e.g.*, seasonal cycles, predator-prey interactions, or habitat loss), then the distinction is empirically significant.

Epidemics of some pathogens are fast enough that demography (reproduction and natural death) may be ignored. In this case, and assuming density dependent transmission, these equations reduce to the *closed SI epidemic*,

$$\frac{dS}{dt} = -\beta SI \quad (5)$$

$$\frac{dI}{dt} = \beta SI. \quad (6)$$

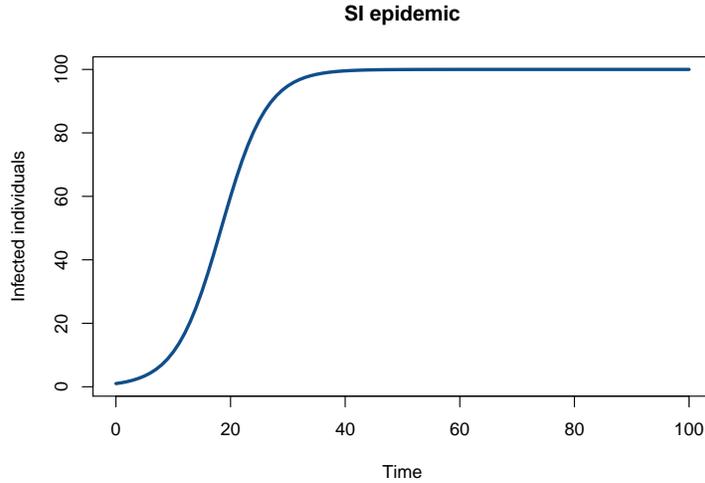


Figure 2: Solution of the  $SI$  epidemic with  $\beta = 0.0025$ ,  $N = 100$ , and  $I_0 = 1$ .

Substituting  $S = N - I$  we have

$$\frac{dI}{dt} = \beta I(N - I), \quad (7)$$

which is a form of the logistic equation, with solution

$$I(t) = \frac{N e^{\beta N t}}{N/I_0 - e^{\beta N t} - 1}. \quad (8)$$

Figure 2 shows a representative trajectory. Equilibria of this model are the *disease free equilibrium*  $(S^*, I^*) = (0, N)$  and the *endemic equilibrium*  $(S^*, I^*) = (N, 0)$ . Only the endemic equilibrium is stable in the case of the closed  $SI$  epidemic.

Things become more complicated once we introduce demography and/or disease-induced mortality. Obviously, if we *only* introduce disease-induced mortality (and not, say, demography) then the population is inevitably exterminated. (Why is this obvious?) Therefore, we will start by introducing demography. For simplicity, we will assume that birth and death rates are equal (the population is neither increasing nor decreasing in the absence of the pathogen) and density-independent. Thus, we have the *open SI epidemic*,

$$\frac{dS}{dt} = bS + bI - bS - \beta SI = bI - \beta SI \quad (9)$$

$$\frac{dI}{dt} = \beta SI - bI. \quad (10)$$

As a result of our assumption that per capita birth and death rates are equal the births and deaths due to susceptibles individuals (terms  $bS$  and  $-bS$  in the first equation) cancel. This leaves a  $bI$  in the flux into the susceptible pool. Interestingly, this is exactly canceled by the death of infected individuals ( $-bI$  in the infected pool). A model in which the gain of susceptibles is exactly equal to the loss of infected individuals might alternatively be interpreted as return of those infected individuals to the susceptible pool. Thus, the open  $SI$  epidemic is also a closed  $SI$  epidemic with recovery. This model is also known as the  $SIS$  epidemic.

Because population size  $N$  is a fixed constant, the  $SIS$  epidemic may also be rewritten as a single equation,

$$\frac{dI}{dt} = \beta(N - I)I - bI, \quad (11)$$

which is analogous to a logistic model with harvesting. Again, the model may be treated analytically, with solution

$$I(t) = \frac{(\beta N - b)I_0 e^{(\beta N - b)t}}{(\beta N - b)\beta I_0 [e^{(\beta N - b)t} - 1]}. \quad (12)$$

Interestingly, however, whereas the simple  $SI$  epidemic proceeded inevitably to 100% prevalence, in the  $SIS$  epidemic the system settles to an *endemic equilibrium* that is number less than  $N$ . This equilibrium may be found by setting equation 11 to zero and solving for  $I$ ,

$$I^* = N - \frac{b}{\beta}. \quad (13)$$

This equation shows that if  $b/\beta > N$  the “endemic equilibrium” is a negative value, which is not a biologically feasible solution. In this case, the pathogen cannot be maintained in

the population (Figure 3). In general, host-parasite models exhibit such criteria that establish the stability of the endemic equilibrium. Often this criterion is expressed in terms of the *basic reproduction number*,  $R_0$ , defined as the number of secondary cases after introduction of the pathogen into a wholly susceptible population. Since the number of secondary cases is greatest when the number of susceptible individuals is greatest (*i.e.*, there is only one infected individual), then the *realized* number of secondary cases per case will only go down as the outbreak progresses. From this, we can deduce that for an outbreak to occur  $R_0$  must be greater than one, which supplies the *invasion criterion* that must be met for a pathogen to establish in a population. For the *SI* epidemic one might say that  $R_0 = \infty$  since an infected individual will continue to transmit to remaining susceptible individuals in the population indefinitely. In the case of the *SIS* epidemic, the number of secondary infections can be determined from the rate at which secondary infections occur  $\beta S = \beta N$  times the time interval over which the individual is infectious  $1/b$ . Thus, for the *SIS* epidemic we have

$$R_0^{SIS} = \frac{\beta N}{b}. \quad (14)$$

We now return to the problem of disease-induced mortality. What happens in the *SIS* epidemic when infection results in a higher rate of mortality? The model is

$$\frac{dS}{dt} = bI - \beta SI \quad (15)$$

$$\frac{dI}{dt} = \beta SI - bI - \alpha I. \quad (16)$$

We cannot make use of our trick of substituting  $S = N - 1$  since  $N$  is no longer a fixed quantity. We can, however, study the invasion criterion. Reasoning as before, we derive the basic reproduction number

$$R_0^{SIS-\alpha} = \frac{\beta N}{b + \alpha}. \quad (17)$$

Under what conditions, then, may the pathogen invade? The formula in equation 17 can be looked at from many points of view. Recalling that invasion may occur if  $R_0 > 1$  and rearranging, we obtain a *host population size threshold*:

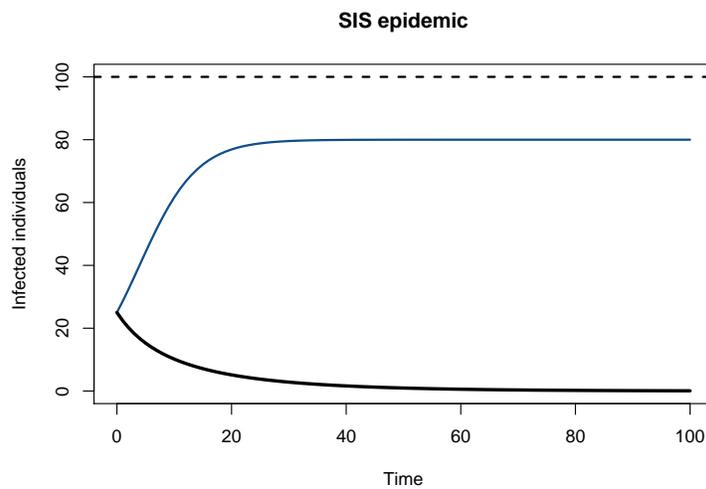


Figure 3: Solution of the *SIS* epidemic with  $\beta = 0.0025$ ,  $N = 100$ , and  $I_0 = 25$ . When birth rate is low ( $b = 0.05$ , blue line) the pathogen achieves prevalence at an endemic equilibrium less than that of the total population size ( $N = 100$ , dashed black line). However, if the birth rate is sufficiently high ( $b = 0.3$ , solid black line) the endemic equilibrium is in the negative quadrant and the pathogen cannot persist.

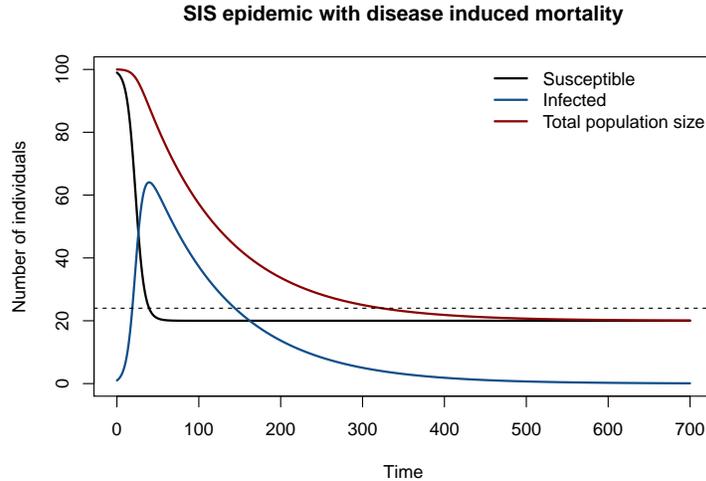


Figure 4: Solution of the *SIS* epidemic with  $\beta = 0.0025$ ,  $N_0 = 100$ ,  $b = 0.05$ , and  $I_0 = 1$ . When the host population is greater than the threshold at  $S_{thresh} = (b + \alpha)/\beta$  (dashed line) the number of infected individuals increases (blue line) and the number of susceptible individuals decreases (black line). Due to disease-induced mortality, the total population size also decreases (red line) reaching an asymptote at a value slightly lower than the invasion threshold.

$$N > \frac{b + \alpha}{\beta}. \quad (18)$$

Equivalently, we predict that a virulent non-immunizing pathogen will reduce a host population to the threshold in equation 18, at which time the pathogen will cease to increase, leading eventually to extinction of the pathogen. Figure 4 shows that this is precisely what happens. Possibly this is what is now happening with the Tasmanian devil.

## Immunizing infections

Up to now we have ignored what is possibly the most remarkable aspect of host-parasite interactions, the evolution of the host immune response. Few pathogens are both virulent and able to avoid a host immune response. Figure 4 illustrates why this is not an evolutionarily

viable strategy. Thus, most pathogens induce an immune response in the host, which may then feed back to affect the subsequent dynamics of the pathogen in the host population. The key difference to the situations considered above is that hosts that have developed specific immunity to a pathogen are neither susceptible nor infectious. What consequences does this have for the dynamics of epidemics, the invasion of novel pathogens, and the regulation of host populations by their pathogens?

To address these questions we will study the *SIR* model, an extension of the *SI* model introduced above with an additional compartment of the recovered (and immune) portion of the population. In the general case (open population with disease induced mortality) the equations are

$$\frac{dS}{dt} = f(N) - dS - \lambda S \quad (19)$$

$$\frac{dI}{dt} = \lambda S - \gamma I - dI - \alpha I \quad (20)$$

$$\frac{dR}{dt} = \gamma I - dR, \quad (21)$$

where  $\gamma$  is the *recovery rate* and  $\gamma^{-1}$  is the *infectious period*. Often we will assume equal constant per capita birth and death rates so  $f(N) = b(S + I + R)$  and  $d = b$ . For the model with density dependent transmission and no disease induced mortality,

$$\frac{dS}{dt} = b(S + I + R) - bS - \beta SI \quad (22)$$

$$\frac{dI}{dt} = \beta SI - \gamma I - bI \quad (23)$$

$$\frac{dR}{dt} = \gamma I - bR, \quad (24)$$

the basic reproduction number is

$$R_0^{SIR} = \frac{\beta N}{\gamma + b}. \quad (25)$$

As in the open *SIS* epidemic, if  $R_0 > 1$  the population will tend to an endemic equilibrium, in this case with the abundance of  $S$ ,  $I$ , and  $R$  are given by the non-zero quantities  $(S^*, I^*, R^*) = (N/R_0, \frac{b}{\beta}(R_0 - 1), \frac{\gamma}{\beta}(R_0 - 1))$ .

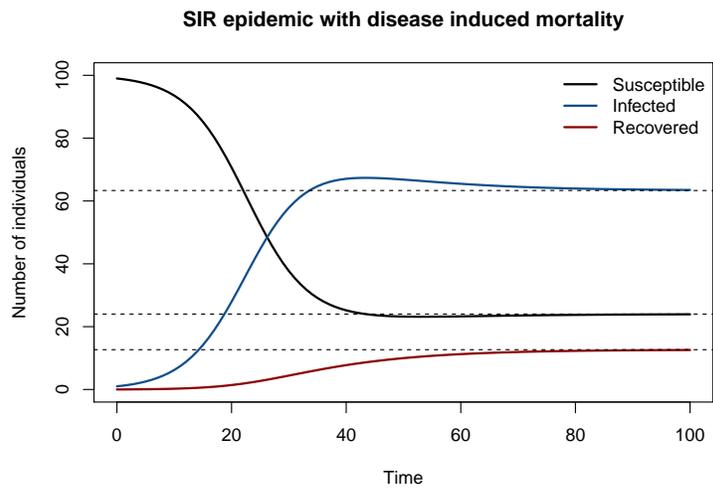


Figure 5: Solution of the *SIR* epidemic with  $\beta = 0.0025$ ,  $N_0 = 100$ ,  $b = 0.05$ ,  $\gamma = 0.01$ , and  $I_0 = 1$ . Equilibrium values shows by the dashed lines.

## Homework exercises

1. Show that equation 5 is equivalent to the logistic equation  $dx/dt = rx(1 - x/k)$ .
2. Why does the host population size decline to a value *lower* than the invasion threshold in Figure 4?
3. Our analysis above suggests that the population of Tasmanian devils may be reduced to a size determined by the transmissibility, birth rate, and disease-induced mortality. Our analysis is based on the premise that transmission is density-dependent. However, since transmission occurs during aggressive encounters related to mating, and these are unlikely to be proportional to population size, it is plausible that transmission is actually frequency dependent. Analyze the *SIS* epidemic with disease-induced mortality and frequency-dependent transmission and determine what the final outcome for the host population is in this case.
4. Obtain the host threshold criterion for the *SIR* epidemic with density-dependent transmission