Homework #1

Using a particular form of the Ricker: $N_{t+1}=N_t e^{r(1-(Nt/K))}$

a) Plot the equaition (aka return map): i.e., N_{t+1} vs. N_t

1) take derivative to find slope (need product rule and chain rule):

product rule: d(fg)/dx=f'g+g'f
chain rule: df(g(x))/dx=f'(g(x))g'(x)
and $d(e^x)/dx=de^x$
so, $d(e^{f(x)})/dx=e^{f(x)} f'(x)$

which leads to (and dropping the t):

$dN_{t+1}/dN_t = 1\ e^{r(1-(N/K))} + N\ e^{r(1-(Nt/K))}\ (-r/K)$
$= e^{r(1-(N/K))}\ (1-Nr/K)$

Evaluate this as N-->0 and as N-->infinite

lim(dN/dN) N-->0 = e^r
lim(dN/dN) N==>inf = ...now we have a problem

the first term on RHS goes to 0 (e^-inf) but the second term has two parts, one of which goes to -infinite and one goes to 0; which one wins?

Use L'Hopital's rule:

$\lim_{x\to c} f(x)/g(x) = \lim_{x\to c} f'(x)/g'(x)$

so, $\lim_{N\to inf}\ N\ e^{r(1-(Nt/K))}\ r/K = \lim_{N\to inf}\ N\ /\ e^{-r(1-(Nt/K))}\ r/K$
$= \lim_{N\to inf}\ 1\ /\ [e^{-r(1-(Nt/K))}\ (-r/K)] = (-)0$

So, we know that our function starts out with slope=e^r and ends with a negative slope that converges to 0.

But we also need to know what the value of our function is at N=0 and N=infinite (or what it approaches as N-->infinite). You should be able to see that N(0)=0 and limN(as N--> infinite)=0.

2) where is the slope 0 (find min and max)?

$0 = e^{r(1-(N/K))}\ (1-Nr/K)$
when N=K/r

is this a min or a max? We know by inspection that it's a max (but we could have taken the second derivative and asked if it's + (min; bowl) or - (min; hill).

So, now we know that the hump occurs at K/r, so as you increase r, the hump moves to the left. If you change K and r by the same multiplier, the hump stays the same.

You should also figure out the value of the function at this hump. Do that by solving the function for N=K/r. You'll see that $N_{t+1} = (K/r)e^{(r-1)}$

The final thing it would be good to know, is where f(N) crosses the 1:1 line, but that's part b (where N=K).

So now we have a good description of the function and we can see how each of these features vary with the parameters of the model, r and K.

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b) Solve for the equilibrium. $N_{t+1}=N_t=N^*$ or:

$N^*=N^*e^{r(1-(N^*/K))}$

$1 = e^{r(1-(N^*/K))}$

take ln of each side...

$0 = r(1-N^*/K)$

$N^* = K$

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c) For what values of r is there a (positive, non-zero) equilibrium?

- The initial slope must exceed 1, so e^r>1, or r>0
- *and K>0 (note K<0 gives odd results and generate positive den-dep)*.
- Note that N=0 also is an equilibrium, for any r.
- Finally, note that r<0, also has an equilibrium at N=0 and N=K. This is an interesting pattern. It's an example of an Allee effect, in which the growth rate is below replacement when N is small, but increases as N increases. Hence, the equilibrium at N=K is unstable. This is an example of positive density dependence.

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d) Is the equilibrium locally stable? Do a stability analysis (as we did with the Taylor series expansion).

evaluate the derivative at N=K

or

$e^{r(1-(K/K))}(1-Kr/K) = 1(1-r) = 1 - r$

So,

Stable if $0<r<2$;

oscillates to equilibrium if $0<r<1$; smooth return if $1<r<2$

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e) Simulate the dynamics for r=2 and K=100 (and r=1.5, 2.1, 2.5)

See the excel file.

f) Do these dynamics match your expectation from e and f?