Homework #1

Using a particular form of the Ricker: Nt+1=Nter(1-(Nt/K))

1. Plot the equaition (aka return map): i.e., Nt+1 vs. Nt

1) take derivative to find slope (need product rule and chain rule):

product rule: d(fg)/dx=f'g+g'f

chain rule: df(g(x))/dx=f'(g(x))g'(x)

and d(ex)/dx=dex

so, d(ef(x))/dx=ef(x) f'(x)

which leads to (and dropping the t):

dNt+1/dNt = 1 er(1-(N/K)) +N er(1-(Nt/K)) (-r/K)

= er(1-(N/K)) (1-Nr/K)

Evaluate this as N-->0 and as N-->infinite

lim(dN/dN) N-->0 = er

lim(dN/dN) N==>inf = ...now we have a problem

the first term on RHS goes to 0 (e^-inf) but the second term has two parts, one of which goes to -infinite and one goes to 0; which one wins?

Use L'Hopital's rule:

limx-->c f(x)/g(x) = lim x-->c f'(x)/g'(x)

so, limN-->inf N er(1-(Nt/K)) r/K = limN-->inf N / e-r(1-(Nt/K)) r/K

= limN-->inf 1 / [e-r(1-(Nt/K)) (-r/K)] = (-)0

So, we know that our function starts out with slope=er and ends with a negative slope that converges to 0.

But we also need to know what the value of our function is at N=0 and N=infinite (or what it approaches as N-->infinite). You should be able to see that N(0)=0 and limN(as N--> infinite)=0.

2) where is the slope 0 (find min and max)?

0 = er(1-(N/K)) (1-Nr/K)

when N=K/r

is this a min or a max? We know by inspection that it's a max (but we could have taken the second derivative and asked if it's + (min; bowl) or - (min; hill).

So, now we know that the hump occurs at K/r, so as you increase r, the hump moves to the left. If you change K and r by the same multiplier, the hump stays the same.

You should also figure out the value of the function at this hump. Do that by solving the function for N=K/r. You'll see that Nt+1= (K/r)e(r-1)

The final thing it would be good to know, is where f(N) crosses the 1:1 line, but that's part b (where N=K).

So now we have a good description of the function and we can see how each of these features vary with the parameters of the model, r and K.

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1. Solve for the equilibrium. Nt+1=Nt=N\* or:

N\*=N\*er(1-(N\*/K))

1 = er(1-(N\*/K))

take ln of each side...

0 = r(1-N\*/K)

N\* = K

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1. For what values of r is there a (positive, non-zero) equilibrium?

* The initial slope must exceed 1, so e^r>1, or r>0
* *and K>0 (note K<0 gives odd results and generate positive den-dep)*.
* Note that N=0 also is an equilibrium, for any r.
* Finally, note that r<0, also has an equilibrium at N=0 and N=K. This is an interesting pattern. It's an example of an Allee effect, in which the growth rate is below replacement when N is small, but increases as N increases. Hence, the equilibrium at N=K is unstable. This is an example of positive density dependence.

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1. Is the equilibrium locally stable? Do a stability analysis (as we did with the Taylor series expansion).

evaluate the derivative at N=K

or

er(1-(K/K)) (1-Kr/K) = 1(1-r) = 1 - r

So,

Stable if 0<r<2;

oscillates to equilibrium if 0<r<1; smooth return if 1<r<2

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1. Simulate the dynamics for r=2 and K=100 (and r=1.5, 2.1, 2.5)

*See the excel file.*

1. Do these dynamics match your expectation from e and f?